Code : 211101

B.Tech 1st Semester Exam., 2014

MATHEMATICS—I

Time : 3 hours — *Full Marks* : 70

Instructions :

(i) *All questions carry equal marks.*

(ii) *There are **NINE** questions in this paper.*

(iii) *Attempt **FIVE** questions in all.*

(iv) *Question No. **1** is compulsory.*

1. Choose the correct answer (any *seven*) :

(a) Value of

$$D^n\left(\frac{1}{ax+b}\right)$$

is

(i) $\frac{n!a^n}{(ax+b)^n}$

(ii) $\frac{(-1)^n n!a^n}{(ax+b)^n}$

(iii) $\frac{(-1)^n n!a^n}{(ax+b)^{n+1}}$

(iv) 0

(b) Value of $D^n\{\sin(ax+b)\}$ is

(i) $a^n \sin(ax+b+n\pi)$

(ii) $b^n \sin(ax+\frac{n\pi}{2})$

(iii) $a^n \sin(ax+b+\frac{1}{2}n\pi)$

(iv) $b^n \sin(ax+b+n\pi)$

(c) The angle of intersection of two curves is defined as the angle between their

(i) normals

(ii) radius vector

(iii) tangents

(iv) None of the above

(d) Pedal equation of the curve $r = ae^{\theta \cot \alpha}$ is

(i) $p = r\sin\alpha$

(ii) $p = r\cos\alpha$

(iii) $p = r$

(iv) $p = r\sin 2\alpha$

(e) A function $f(x)$ has maximum value at $x = c$, if

(i) $f'(c) = 0$ and $f''(c) > 0$

(ii) $f'(c) = 0$ and $f''(c) < 0$

(iii) $f'(c) = 0$ and $f''(c) \neq 0$

(iv) $f'(c) \neq 0$ and $f''(c) < 0$

(f) Let A and B be real symmetric matrices of size $n \times n$. Then which of the following is true?

(i) $AA' = I$

(ii) $A = A^{-1}$

(iii) $AB = BA$

(iv) $(AB)' = BA$

(g) If A is an orthogonal matrix, then A^{-1} is equal to

(i) A

(ii) A^T

(iii) A^2

(iv) None of the above

(h) The value of $B(m, m)$ is

(i) $2^{1-2m} B(m, \frac{1}{2})$

(ii) $2^{1-2m} B(m+1, \frac{1}{2})$

(iii) $2^{1-2m} B(m+\frac{1}{2}, 1)$

(iv) $2^{1-2m} B(m, \frac{3}{2})$

(i) If $D \equiv \frac{d}{dz}$ and $z = \log x$, then the differential equation $x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = 6x$ becomes

(i) $D(D-1)y = 6e^z$

(ii) $D(D-1)y = 6e^{2z}$

(iii) $D(D+1)y = 6e^2 z$

(iv) $D(D+1)y = 6e^z$

(j) erf(∞) is

(i) -1

(ii) $-\infty$

(iii) 1

(iv) 0

2. (a) If $y = a\cos(\log x) + b\sin(\log x)$, then prove that

$$x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$$

(b) Obtain $\tan^{-1} x$ in powers of $(x-1)$.

3. *(a)* If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, then show that

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}$$

(b) If

$$u = \sin^{-1}\frac{x^2 + y^2}{x + y}$$

then show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$$

4. *(a)* Find

$$\lim_{x \to 0}\left(\frac{\tan x}{x}\right)^{1/x}$$

(b) Find the equations of the tangent and normal at $\theta = \pi/2$ to the curve $x = a(\theta + \sin\theta)$, $y = a(1 + \cos\theta)$.

5. *(a)* Using elementary row transformation, find the inverse of the following matrix :

$$\begin{bmatrix} 5 & -1 & 5 \\ 0 & 2 & 0 \\ -5 & 3 & -15 \end{bmatrix}$$

(b) Find the rank of the following matrix by reducing to normal form :

$$\begin{bmatrix} 1 & 2 & -1 & 3 \\ 3 & 4 & 0 & -1 \\ -1 & 0 & -2 & 7 \end{bmatrix}$$

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6. *(a)* Find the eigenvalues and eigenvectors of the following matrix :

$$\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$$

(b) Verify Cayley-Hamilton theorem for matrix A and hence find A^{-1} and A^4 :

$$A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

7. *(a)* Solve (any *two*) :

(i) $\sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0$

(ii) $\frac{dy}{dx} = 1 + \tan(y - x)$

(iii) $2x^2 y \frac{dy}{dx} = \tan(x^2 y^2) - 2xy^2$

(b) Solve :

$$\frac{dy}{dx} = \frac{y + \sqrt{x^2 + y^2}}{x}$$

8. Solve :

(i) $\frac{dy}{dx} + 2xy = 2e^{-x^2}$

(ii) $\frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 8y = 0$

9. (a) Prove that

$$\boxed{m}\ \boxed{m+1/2} = \frac{\sqrt{\pi}\,\boxed{2m}}{2^{2m-1}}$$

(b) Evaluate

$$\int_0^{\pi} \frac{dx}{a+b\cos x}$$

where $a>0$, $|b|<a$

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